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On the universality of global modes in low-density axisymmetric jets

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Experiments conducted in axisymmetric low-density jets reveal that the transition to global instability and the frequency selection of the global mode depend on the operating parameters of density ratio, momentum thickness, and Reynolds number. The onset of the global mode was mapped in the Reynolds number–momentum thickness operating domain for density ratios $S = \rho_j/\rho_\infty$ ranging from 0.14 to 0.5. The results provide convincing evidence of the universality of global oscillations in low-density jets and indicate that conditions in the jet exit plane are responsible for driving the global instability.

1. Introduction

The presence of global instability in closed hydrodynamic systems is well known – examples include Taylor–Couette flow, Rayleigh–Bénard convection, and temporal shear layers – but the presence of intrinsic global modes in open hydrodynamic systems is less well understood. Our current understanding of these modes has come, in part, through experimental observations of the Kármán vortex street at low Reynolds numbers (Provansal, Mathis & Boyer 1987; Sreenivasan, Strykowski, Olinger 1987), the near fields of low-density jets (Sreenivasan, Raghu & Kyle 1989; Monkewitz *et al.* 1990; Yu & Monkewitz 1993; Raynal *et al.* 1996) and countercurrent shear layers (Strykowski & Niccum 1991, 1992), and in part by theoretical work that has provided tangible connections between local stability concepts and the global flow response (Chomaz, Huerre & Redekopp 1991; Monkewitz, Huerre & Chomaz 1993; Pier & Huerre 2001; Chomaz 2005).

Establishing scaling laws for the Kármán vortex street has been an active research area at least since the time of Roshko (1954). Studies aimed at addressing the universality of this scaling were taken up more recently in the careful experiments of Williamson(1988) among many others. The expectation that a so-called universal relationship might be forthcoming was driven in large measure by the understanding that the oscillatory response of the two-dimensional wake flow, at least at low Reynolds numbers, was shown to be described by a Hopf bifurcation (Provansal *et al.* 1987; Sreenivasan *et al.* 1987). Agreement among researchers on the onset of vortex shedding and the Strouhal–Reynolds number relationship lies ultimately in the globally unstable nature of this flow and hence its insensitivity to external background forcing.

Much of the early experimental work on low-density jets focused on establishing the existence of unstable global modes (Sreenivasan *et al.* 1989; Monkewitz *et al.* 1990). These studies documented the spectral content of the jet oscillations and the flow response to forcing to show that the global mode was unchanged by moderate levels of external excitation. Sreenivasan *et al.* showed the existence of unstable global modes in the Mach number/density ratio plane through noting the emergence of a narrow spectral peak. They observed the onset of global instability at a density ratio of approximately $S = 0.6$ at a Mach number of 0.15. Interestingly, they reported that the critical density ratio decreased at Mach numbers above and below 0.15. The low-Mach-number departure was associated with the small diameter of their jet and generally dismissed, though the results presented below will clearly show the parameters responsible for the departure.

Monkewitz *et al.* (1990) conducted a comprehensive study of the global mode in low-density jets by heating the jet gas relative to the ambient fluid. They showed the same type of behaviour as described by Sreenivasan *et al.* (1989) as well as the existence of another unstable mode (called Mode I) that had a higher critical density ratio of approximately 0.73. The second mode (which they referred to as Mode II) had a critical density ratio of approximately 0.63, agreeing quite well with the conclusion of Sreenivasan *et al.* A sharp discrepancy is noted between the work of Monkewitz *et al.* and that of Sreenivasan *et al.* in that the critical velocities are reported as being a factor of three lower. Monkewitz and coworkers conjectured that the momentum thickness was responsible for the apparent disagreement. The current work will show that this assessment was partly true, though the detailed transition is a more complicated function of both the momentum thickness and the Reynolds number taken as independent parameters. Monkewitz *et al.* did look briefly into the frequency scaling of the global mode but no conclusions as to the universality of such data were provided. As they stated, the scaling was based on the "dimensional jet velocity to express our ignorance about the true controlling parameter(s)".

It was not until the work of Kyle & Sreenivasan (1993) that a systematic experimental approach was undertaken to address the frequency scaling issue in low-density jets. The work considered not only the density ratio but also the momentum thickness to be important controlling parameters. Kyle & Sreenivasan concluded that the critical density ratio for an axisymmetric low-density jet was 0.6 and occurred at a value of momentum thickness such that $D/\theta_0 \sim 83$, where D is the jet diameter and θ_0 is the shear-layer momentum thickness in the jet exit plane. Though they considered the independent role that the Reynolds number might play in the scaling, they concluded that the frequency scaled uniquely on the density ratio and momentum thickness over the range of conditions considered in the study.

While the physical mechanisms responsible for global instability of open flows may not be fully understood from experimental observation, theoretical advances have provided substantive connections between the stability of the base flow and the nonlinear flow response. Employing local linear theory, Koch (1985), Huerre & Monkewitz (1985, 1990), Monkewitz *et al.* (1993), Chomaz *et al.* (1991), among others, determined that a necessary condition for the emergence of a linear global mode was a finite region of locally absolutely unstable flow. Koch conjectured that frequency selection in the absolutely unstable wake was determined by the streamwise profile at the boundary between absolutely unstable flow upstream and convectively unstable flow downstream. Monkewitz *et al.* (1993) and Monkewitz & Nguyen (1987) suggested that the global frequency was determined by the upstream most absolutely unstable profile in the wake. Hammond & Redekopp (1997) found reasonable agreement between the frequency of the saddle-point mode and that revealed by direct numerical simulation in wakes.

In all these cases, the frequency selection criteria were based on the emergence of linear global modes. Pier & Huerre (2001) embarked on a comprehensive examination of nonlinear global modes and the conditions under which disturbances will lead to global flow resonance (see also the review by Chomaz 2005). By studying the nonlinear dispersion relation of a slowly diverging wake, Pier & Huerre conclude that the real wake frequency is selected by the first streamwise station of non-negative absolute growth, a conjecture supported by their numerical simulations and the earlier results of Monkewitz & Nguyen (1987). More recent findings of Lesshafft *et al.* (2005) explore this connection in the context of low-density axisymmetric jets, by conducting both direct numerical simulations (DNS) and spatio-temporal stability calculations on the computed base flow. The frequency predictions of the linear theory were substantially lower than those found in the DNS, a point noted by the authors as possibly being associated with the high Reynolds number of the simulation; a reference is made to a forthcoming publication that suggests better agreement when the Reynolds number is reduced.

Global modes have been shown to exist in low-density jets over a wide range of operating conditions, but there remain unanswered questions regarding the universality of the flow response. The current work aims to shed light on this issue, by carefully examining the behaviour of low-density jets using a facility capable of independently varying the jet density ratio, momentum thickness and Reynolds number over a substantial operating domain. The general procedure was to use helium or combinations of helium and nitrogen to create a low-density gas jet. The jet facility consisted of a nozzle followed by variable-length extension pieces to develop the boundary layer and hence the initial momentum thickness of the jet. In the sections below, the facilities will be described, followed by a discussion of the jet operating conditions, the identification of global modes, and finally a summary of our global mode frequency scaling.

2. Facilities and instrumentation

Variable-density jets were created using commercially available mixtures of helium and nitrogen to create precise density ratios $S = \rho_j/\rho_\infty$ between 0.14 and 0.5. The primary gas was passed through a plenum chamber containing honeycomb and screens used for flow conditioning. The flow was then accelerated through an aluminum fifth-order polynomial nozzle with an area contraction ratio of 100. The exit of the nozzle was machined to receive variable-length brass extension tubes. The extension tubes were precisely bored out to match the nozzle exit diameter ($D = 0.635$ cm). The outside of the extension tubes had a third-order polynomial machined onto the last $1.5D$ to reduce the outer dimension from $2D$ to $1D$ and thereby produce a fine knife edge at the nozzle lip.

Schlieren flow visualization was employed using two first-surface f5 15 cm primary mirrors set up in the classic Z-type configuration. A halogen automotive headlamp was used as the light source and a Nikon D-70 camera with a Nikor 105 mm micro lens was used to capture the images, typically at a shutter speed of 1/8000 s. No post-processing was done other than converting the images into greyscale.

Hot-wire anemometry was the primary quantitative diagnostic used to characterize the jet response. The probe was a Dantec model 55P11 straight hot-wire probe, used in conjunction with a Dantec constant-temperature anemometer. Frequency data were collected using a personal computer connected to a 16-bit converter and an uncalibrated hot-wire. One-second data sets were captured at a sampling rate of 5 kHz for the majority of situations, but up to 50 kHz when the primary frequencies were over 2.5 kHz. Data reduction was accomplished via Matlab algorithms for Fourier

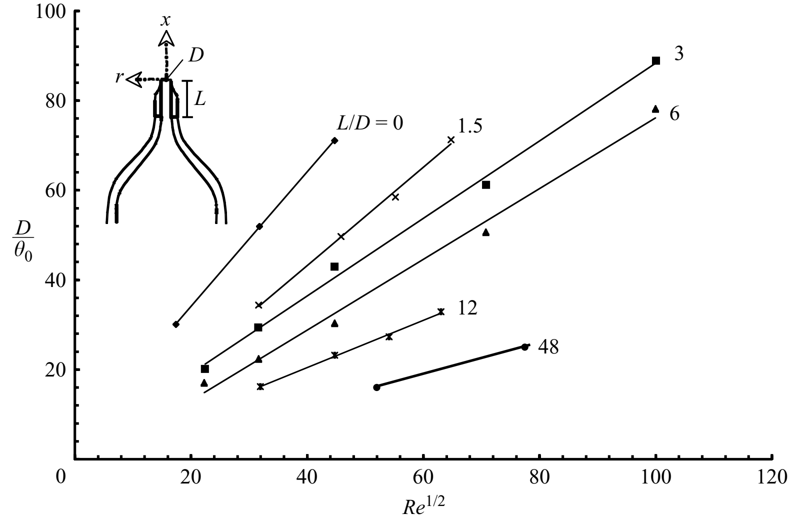


FIGURE 1. Facility calibration curves.

transforms and fluctuation measurements. FFTs are ensemble averaged over 5–7 data records to reduce variance. A hot wire calibrated in air was used to measure the shear-layer momentum thickness immediately downstream of the jet exit.

3. Operating domain and jet initial conditions

The current facility was designed to achieve operation in the Re–D/θ_0 domain over a considerably larger range of conditions than had been previously attempted. For initially laminar separated shear layers, the momentum thickness varies as the square root of the Reynolds number. To decouple Re and D/θ_0, carefully machined extension tubes were employed to allow boundary layer development prior to the jet exiting into the quiescent ambient. To provide sufficient separation in Re and D/θ_0 extension tubes were created of lengths $L/D = 0$, 1.5, 3, 6, 12, and 48. For any particular configuration, the Re–D/θ_0 dependence was established using pure air as the primary gas. For each extension tube the velocity profiles were measured at the exit plane using a calibrated hot wire; the profile was integrated from the centre of the jet to the location where the velocity dropped below 20 % of the centreline velocity when determining θ_0.

The resulting calibration curves are provided in figure 1; the inset to the figure provides the experimental layout and coordinate frame. The square-root dependence on Re corroborates that the boundary layers exiting the tubes are laminar over the range of conditions reported. The Reynolds number for the current study was calculated using $Re = \rho_j U_j D/\mu_j$ where the subscript j refers to the jet centreline conditions. Gas mixture viscosities were determined using the Wilke (1950) formulation. The centreline turbulence intensity for the facility was less than approximately 0.2 % of the centreline velocity and was found to be independent of Reynolds number and extension tube for the range of conditions reported in this study. The background turbulence was broadband with no distinct frequencies dominating the spectrum suggesting no facility instability or resonance was present. Mach numbers for the current results were below 0.1 and Richardson numbers were less than 0.005 and generally significantly less; neither compressibility nor buoyancy is believed to be

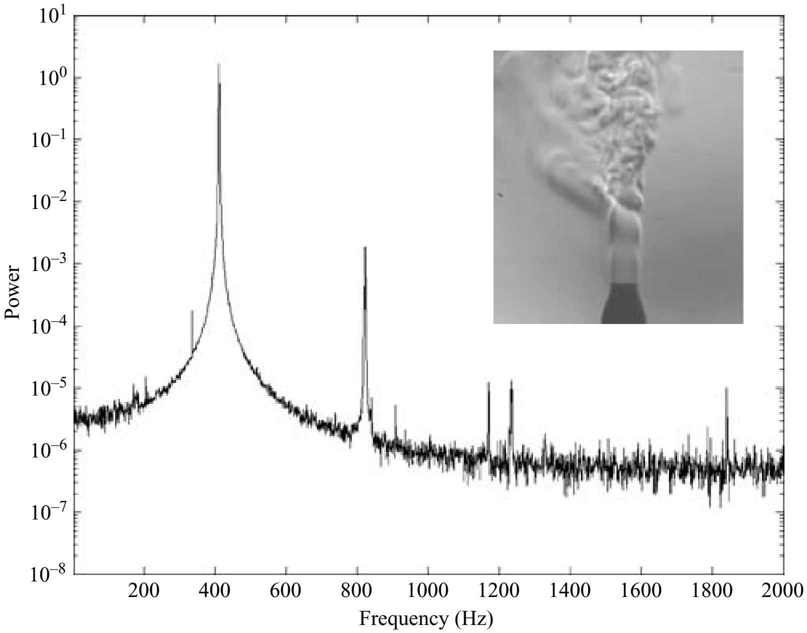


FIGURE 2. Hot-wire power spectrum and flow visualization of a globally unstable jet at $S = 0.14\,(L/D = 1.5,\ Re = 650)$.

important factors for the current findings. It is also important to point out that any universality exhibited in our results would pertain to low-density jets possessing steep density profiles (top hat in the current study) at the nozzle exit plane. Raynal *et al.* (1996) have clearly shown theoretically that the flow stability is sensitive to the shape and alignment of the density field and velocity field. Finally the independent effects of shear-layer viscosity ratio and Schmidt number were not considered in this study and hence must be considered as uncontrolled variables.

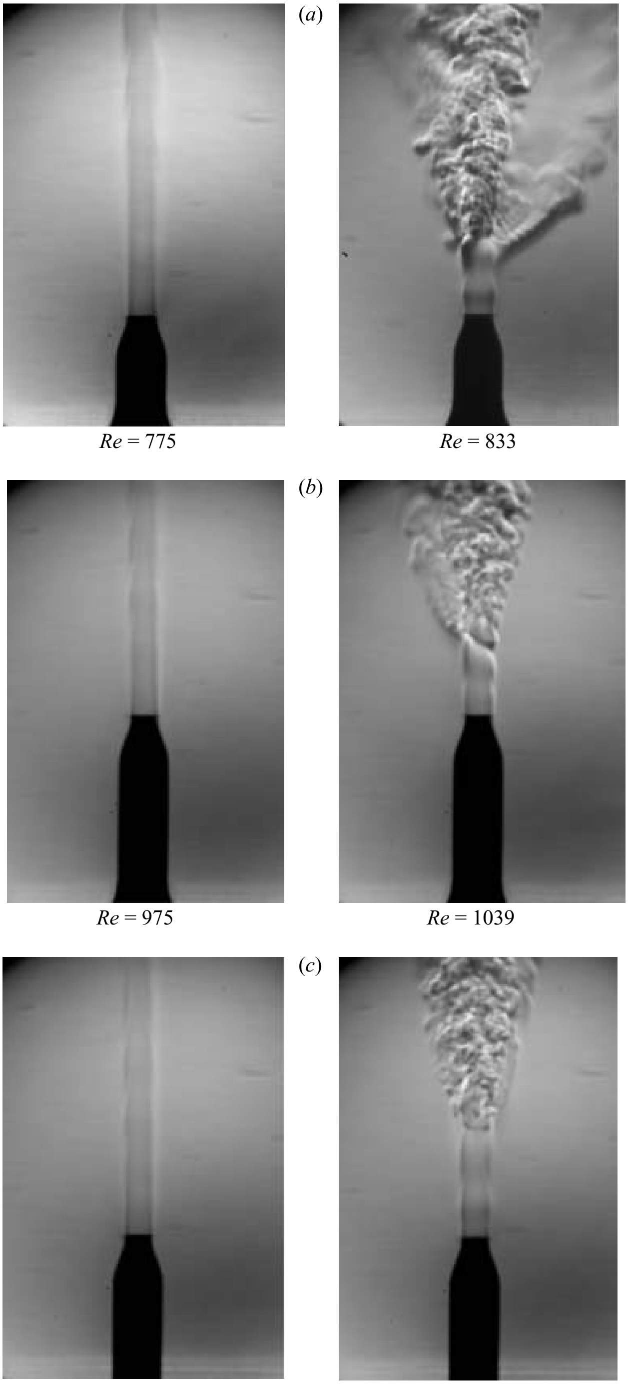
4. Unstable global modes and their onset

Previous research has shown that low-density jets, which are not initially turbulent in nature (e.g. see Pitts 1991), become globally unstable and are characterized by several well-defined features such as pure-tone oscillations, dramatic spreading, the presence of side jets, and insensitivity to external excitation. Schlieren imaging was used to qualitatively observe these flow features and confirm the strong axisymmetry of the vortex formation process in the jet near field under all conditions studied. Figure 2 shows the spectral composition of a globally unstable jet at $S = 0.14$ and $Re = 650\,(L/D = 1.5)$ along with a representative photograph under the same conditions. The photograph illustrates the axisymmetric nature of the initial vortex development, which typically persists for a few jet diameters downstream prior to a rapid breakdown; in figure 2 the conditions downstream of vortex breakdown display side jets as observed by Monkewitz *et al.* (1990).

Given the existence of the global mode, a systematic investigation into the transition was made to determine the critical conditions for each configuration and density ratio. This was largely achieved by examining hot-wire records – where the onset of global modes were concomitant with the rapid emergence of pure-tone oscillations in the signals – as well as simultaneously studying the schlieren images to confirm the initial axisymmetry of the near-field global mode. In practice, the onset of global instability was found at a particular density ratio by slowly increasing the Reynolds number for a fixed extension tube, thereby traversing the Re–D/θ_0 space on a curve unique to that configuration; the possibility of hysteretic effects was also explored by decreasing the Reynolds number after global instabilities were identified. Each nozzle–tube combination possessed a distinct transition, which can be qualitatively observed in figure 3. The images reveal the flow behaviour on either side of the transition for three of the facilities at $S=0.14$; the left-hand images correspond to spatially unstable but globally stable flows and the right-hand images reveal the flow response post transition. While details of the transition depend subtly on L/D, the primary features of global instability are common in all facilities. These include: the rapid growth of pure-tone oscillations associated with initially axisymmetric near-field structures; the abrupt termination of the potential core; and the rapid three-dimensional breakdown of the jet. The existence of side jets was not universal over the range of conditions studied. Owing to the dramatic amplification of the oscillations above transition, hot-wire records can be conveniently used to document the operating parameters corresponding to the onset of global instability in each configuration. Figure 4 summarizes the strength of the oscillations (recorded on the jet centreline at $x/D=1.5$) as the Reynolds number was varied around the transition for $S=0.14$ and 0.27 at several L/D. The amplitude of the RMS velocity fluctuations squared typically increases by more than an order of magnitude over a narrow range of Reynolds numbers; this distinct transition is consistent with those observed by Monkewitz *et al.* (1990) and Kyle & Sreenivasan (1993). The hot wire was positioned on the jet centreline at $x/D=1.5$ to ensure that it remained in the unmixed core gas and at a location where the presence of the probe was determined not to alter the flow response.

The transitions apparent in figure 4 were used to establish stability boundaries in the (Re, D/θ_0)-plane for each density ratio, which are shown in figure 5; data not presented at $S=0.5$ and for $L/D=48$ are consistent with those shown in figure 4. An example trajectory ($L/D=6$) is included in figure 5 as the dot-dashed curve. The solid lines demark global stability boundaries at each density ratio; generally the conditions to the left of each boundary are globally stable and to the right globally unstable. However, the stability boundary at $S=0.5$ turns back on itself, where the flow is globally unstable within the curve and globally stable outside; the presence of nearly closed stability curves was also described by Kyle & Sreenivasan (1993) in the ($S, D/\theta_0$)-plane. The upper portion of the stability boundary at $S=0.5$ is dashed to indicate that, while the global mode is turned off, the transition is not as distinct as that on the lower boundary. Caution should be exercised in interpreting the stability boundaries outside the range of conditions studied. For instance, in experiments conducted in the facility at $L/D=0$, global instability was not observed at any Reynolds number for a density ratio of 0.5, as this facility trajectory falls below the closed curve positioned above it at $S=0.5$. This would imply that the critical density ratio for the facility with $L/D=0$ must lie somewhere between 0.27 and 0.5. Preliminary measurements made at significantly higher Reynolds numbers suggest that stability boundaries at $S=0.14$ and 0.27 also turn upward, but those transitions are also less distinct and will require further exploration.

It is clear that the emergence of the oscillating global mode in low-density jets will be observed over a wide range of Re and D/θ_0 depending on the specific nature of



$Re = 775$ $Re = 833$

$Re = 975$ $Re = 1039$

$Re = 1375$ $Re = 1450$

FIGURE 3. Schlieren flow visualization of globally stable (left) and globally unstable (right) jets at $S = 0.14$ for (*a*) $L/D = 3$, (*b*) $L/D = 6$, and (*c*) $L/D = 12$.

the facility used to create the initial conditions of the jet. Although we did not study density ratios above 0.5, both theory (Monkewitz & Sohn 1988) and experiments (Kyle & Sreenivasan 1993) suggest an upper bound on the density ratio needed to

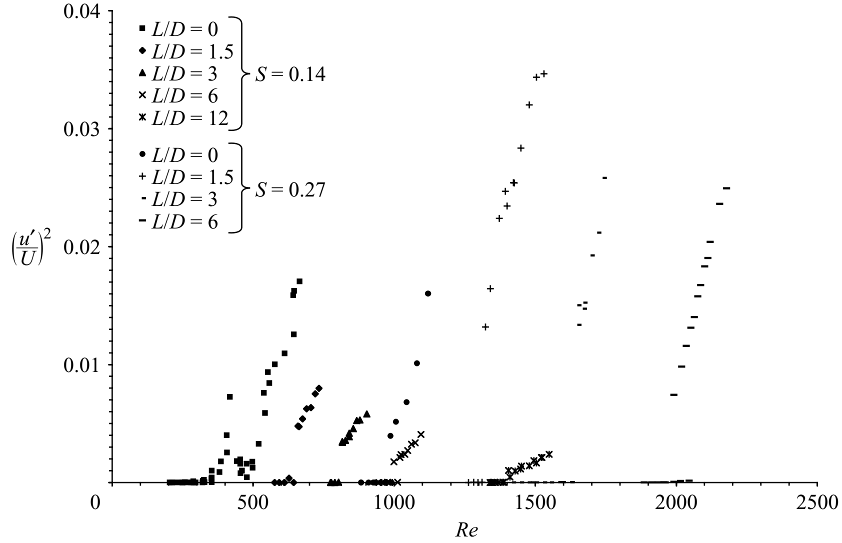


FIGURE 4. Square of RMS velocity fluctuations measured on the jet centreline at $x/D = 1.5$.

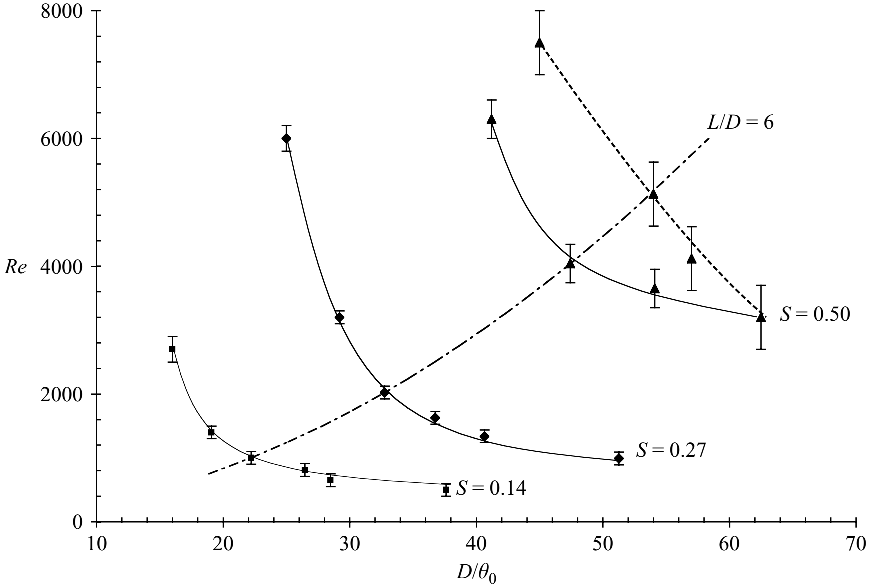


FIGURE 5. Global transition for density ratios of 0.14, 0.27 and 0.5.

support global instability ranging between approximately 0.6 and 0.73. While we do not have the evidence to propose an absolute maximum, experiments designed to evaluate this condition will need to be capable of subtle manoeuvring in the (Re, D/θ_0)-space to avoid underprediction. Assuming that the stability boundaries are unique at each density ratio, the maximum density ratio capable of supporting global modes would be found in the (Re, D/θ_0)-plane within the curve defined by the transition at $S = 0.5$, thereby helping to narrow the operating space over which to search.

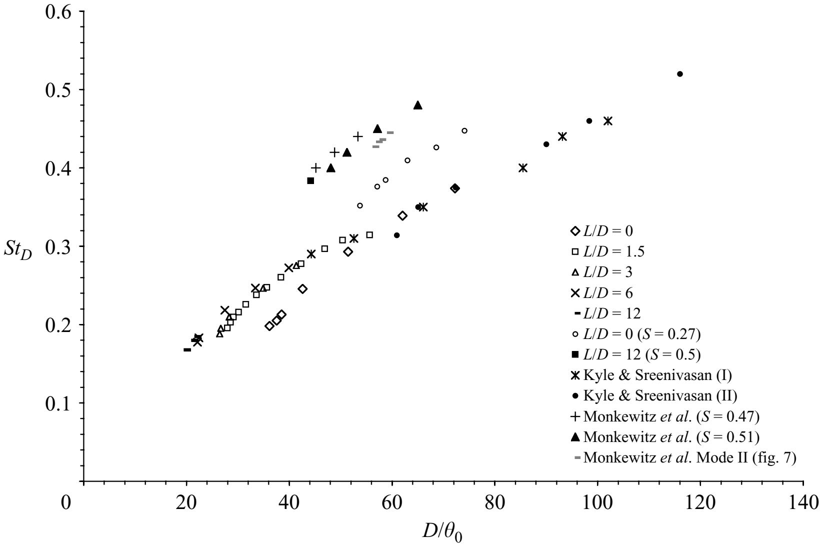


FIGURE 6. Strouhal-number scaling of global instability in low-density jets.

5. Universal frequency scaling

Proposing universal scaling laws in spatially unstable flows requires an understanding of the dimensionless parameters governing the flow physics, but is complicated by the insidious nature of inherent forcing, e.g. uncontrolled background turbulence. While a careful laboratory investigation may successfully minimize these inherent disturbances, they nonetheless are present at some level and potentially over myriad spatial wavelengths. The beauty of global instability – as least as it is observed in the absence of excessive external excitation – is its insensitivity to this noise and hence why we can hope that a universal relationship may be identified if the key underlying parameters can be found. Williamson (1988) took this approach in attempting to unravel the universal frequency behaviour of the classic Kármán vortex street at low Reynolds numbers, a problem plagued by flow non-uniformity and cylinder end conditions. Our examination of low-density axisymmetric jets reveals that the onset of global oscillations depends on S, D/θ_0 and Re, as long as compressibility and buoyancy effects do not come into play. It is the expectation that nature must be bound by this space that strongly suggests a universal frequency law for low-density jets can be found.

The primary frequency of the globally unstable mode is characterized by a distinct peak in the spectrum, is a value of practical interest, and for our experiments here, is quite easily measured. As a first attempt to scale the frequency data, the conclusion of Kyle & Sreenivasan (1993) was employed, namely that the non-dimensional global frequency ($St_D = fD/U$) is a unique function of density ratio and momentum thickness. Figure 6 shows data for the current facility along with that of Kyle & Sreenivasan (1993) and Monkewitz *et al.* (1990). At any particular density ratio the scaling is unsatisfactory, though improves slightly with increasing D/θ_0. The data overall suggest a departure due to uncontrolled variable(s).

Assuming that the global instability is responsible for a periodic travelling wave, the frequency can be non-dimensionalized via inertial or viscous time scales. Given the inability of St_D to fully capture the physics in the present experiments and the insight gleaned from the dependence of transition on Re and D/θ_0, we choose to normalize the frequency by the viscous time scale D^2/ν; in so doing the Reynolds number is retained in the frequency dependence. It is conjectured that the convective velocity of the travelling wave will be a function of velocity ratio and density ratio. For the current problem, the velocity ratio is not varied, reducing the convective velocity to a function of density ratio alone, resulting in

$$\frac{fD^2}{\nu} = Re\frac{D}{\lambda}g(S).$$

While the precise scaling of λ is unknown in the current problem, it is believed that curvature may be an important factor in the flow dynamics owing to the strong streamwise curvature set up in the separated shear layer by the pulsatile nature of the jet column, as consistently seen in near-field imaging of the jet (e.g. see figures 2, 3). The role of centripetal acceleration enters through one-half powers, such as in Taylor–Görtler and Taylor–Couette flows (Schlichting 1979) and pressure-driven channel flows (Dean 1928). Therefore we anticipate that D/λ may scale as $(D/\theta)^{1/2}$. The appropriate choice of where to measure θ is a matter of both academic and practical concern; the choice was made based on several factors. In the laboratory the momentum thickness is a function of streamwise distance and in the present problem measuring $\theta(x)$ is complicated by the concentration gradients present in the separated shear layer and was not undertaken; it is also important to note that experimental literature typically employs the profile in the jet exit plane. Furthermore, at the nozzle exit there exists a step in the density profile coincident with a maximum in the derivative of the velocity profile; this combination should yield the least-stable position in the flow based on the work of Raynal *et al.* (1996). Lastly, Pier & Huerre (2001) suggest that the global frequency will be imposed by the first streamwise profile having a non-negative absolute growth. Hence selecting the momentum thickness in the jet exit plane satisfies the practical issues, while also providing an opportunity to examine the frequency selection mechanism proposed by Pier & Huerre.

By examining the inviscid calculations performed by Pavithran & Redekopp (1989) in variable-density planar shear layers and Jendoubi & Strykowski (1994) for axisymmetric low-density jets, one observes that the frequency scales approximately as the square root of the density ratio over the range of S considered in this study. Viscous stability calculations performed by Lesshafft *et al.* (2005) provide further insight into the functional dependence of frequency on density ratio for axisymmetric heated jets. Over a limited range of density ratios they show that the absolute frequency of the inlet profile scales more strongly with S than the half-power, though somewhat less than linearly. Using theory as a starting point, a satisfactory functional relationship for density ratio was ultimately determined by trial and error. The final relationship can be seen in figure 7 where data from several experimental groups and covering a large range of operating parameters have been collapsed onto a single universal curve.

The scaling reveals a linear relationship between fD^2/ν and the abscissa, namely $fD^2/\nu = A + B\,Re(D/\theta_0)^{1/2}(1 + S^{1/2})$; the inset is zoomed-in to the smaller values of the abscissa. A least-squares linear fit to the present experimental results yields $A = -37$ and $B = 0.034$. This relationship is shown as the solid line in figure 7; the dashed line is an extension of the relationship beyond the present data, but illustrates that the fit is quite good when compared to other studies at higher values of the abscissa. The points from Kyle & Sreenivasan (1993) correspond to the oscillating mode (Kyle I and II correspond to the 13.3 mm and 9.3 mm nozzles respectively)

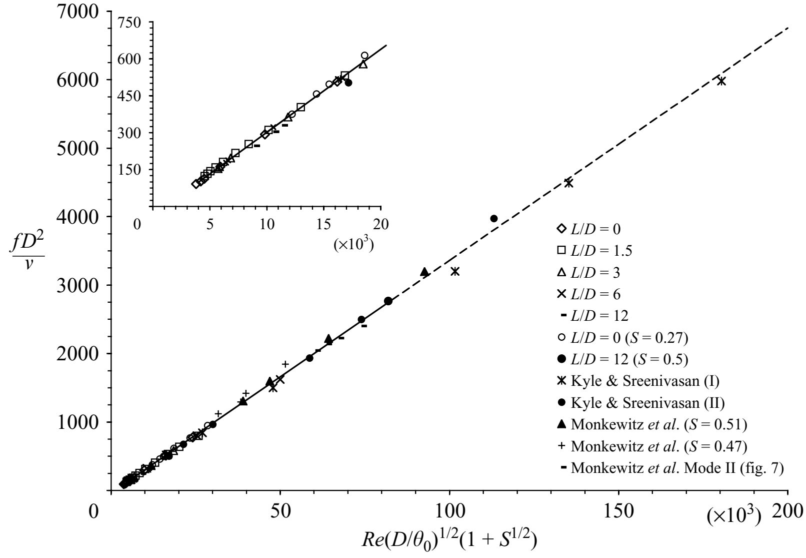


FIGURE 7. Universal scaling of the primary frequency of a globally unstable axisymmetric jet. (Inset is a close-up of the low values of the abscissa and ordinate.)

while the points from Monkewitz *et al.* (1990) correspond to their Mode II. Kyle & Sreenivasan and Monkewitz *et al.* concluded that these modes corresponded to the same global instabilities in their jets and they agree with the scaling of the present data remarkably well, confirming the universality of the global mode. The subtle differences between data sets are likely to be due to uncertainties in the momentum thickness, which for the current work was of the order of a couple of percent. Care must be taken when comparing to other data because of inconsistencies in defining operating conditions. Data from other studies were converted using hyperbolic tangent velocity profiles to obtain consistent definitions of momentum thickness. Similarly, the Reynolds number definition of Monkewitz *et al.* was converted to that defined in this paper (i.e. using the viscosity of the jet core, not the average viscosity between the jet and ambient fluids). Once these conversions are made the collapse seen in figure 7 was obtained. The scaling validates the importance of the Reynolds number in addition to D/θ_0 and S in capturing the physics of global oscillations in low-density jets having initially steep density profiles.

Previous studies on low-density jets have reported a number of unstable modes in addition to the so-called oscillating mode on which the focus of this paper has been thus far. These modes have not been fully explored, but are typically associated with secondary peaks in the power spectra and are often somewhat less discrete in nature; they have been observed both with and without the presence of the oscillating mode. Studies by Monkewitz *et al.* (1990), Kyle & Sreenivasan (1993), Russ & Strykowski (1993), and Russ, Strykowski & Pfender (1994) report these modes, and their results as well as those in the present facility have been recast using the proposed scaling. The scaling, shown in figure 8, adequately captures the reported frequencies, though with arguably higher scatter than the oscillating mode itself. This could be due, in part, to the fact that these modes are not as spectrally narrowband as the primary oscillating mode. But Mode I reported by Monkewitz *et al.* is shown to collapse well on the lowermost curve and was spectrally rather narrow and identified by the authors as

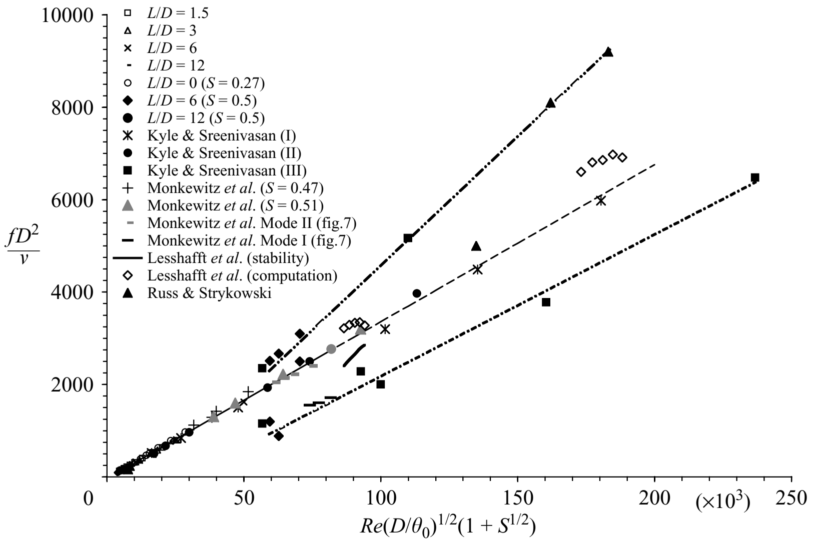


FIGURE 8. Universal scaling of primary and secondary frequencies of low-density axisymmetric jets.

a global mode. Lesshafft *et al.* (2005) concluded that Mode I of Monkewitz *et al.* is not the consequence of a global mode, as the base flow does not support absolute instability. A close examination of figure 8 indicates that the lower curve is essentially one-half of the upper, indicating that the data reflect an unstable mode and its subharmonic due to vortex pairing. It seems plausible that the modes emerging with increasing abscissa in figure 8 are simply convectively unstable shear layer modes, which have long been known to scale initially as $f\theta/U = 0.012$, or subharmonics thereof (Monkewitz & Huerre 1982).

The fact that Monkewitz *et al.* (1990) identified Mode I as a global mode may have been the consequence of several factors. First, Mode I was observed to emerge at a critical density ratio of 0.73, which is remarkably close to with the theoretical prediction of 0.72 (Monkewitz & Sohn 1988). Second, the mode was quite narrowband, providing at least circumstantial evidence of a global instability. Finally, it is possible that Mode I was the consequence of a marginal global instability. While Lesshafft *et al.* (2005) did not find evidence for local absolute instability in their particular flow field, the studies of Raynal *et al.* (1996) clearly illustrate the importance of subtle variations in the shape and alignment of the velocity and density fields. Yu & Monkewitz (1990) show that the evolution of identical velocity and density profiles can yield locations outside the exit where absolute instability can exist. Therefore it seems plausible for a flow to be initially stable and develop farther downstream into an absolutely unstable combination of velocity and density profiles, via differential diffusion of momentum and mass/thermal diffusion. Such scenarios, while perhaps academic, could be established in the laboratory by a clever choice of Schmidt or Prandtl number.

Also included in figure 8 are results from Lesshafft *et al.* (2005) where they computed the frequencies from unstable heated jets both via DNS computations and using viscous linear stability theory. The computations and stability calculations agree reasonably well with the current scaling of the global mode, though the DNS and stability predictions are above and below our scaling, respectively. The fact that the present experimental findings collapse so well is strong support for the theoretical underpinnings of Pier & Huerre (2001), but does not help resolve why the self-consistent approach of Lesshafft *et al.* does not support the first streamwise non-negative absolute mode as being responsible for the jet's frequency selection. Lesshafft *et al.* do indicate some potential problems with their simulations at higher Reynolds numbers, and allude to forthcoming results at lower Reynolds numbers, which may shed additional light on this matter.

Finally figures 7 and 8 imply that there may be critical values of the abscissa needed to observe the emergence of these modes, whether primary or secondary, much in the same spirit that a universal critical Reynolds number is associated with the vortex shedding process. But the inset to figure 7 would appear to dispel this thought, since the lower bound on the universal curve appears to depend on the extension length, achieving its lowest level for $L/D=0$. While $L/D=0$ might seem to be an attractive end point, the reader is reminded that the facility trajectory depends both on the length of the extension and the shape of the contraction itself, namely the $L/D=0$ curve is only unique for our particular nozzle contraction.

6. Summary and conclusions

The current work investigates the universal nature of the so-called oscillating mode in low-density jets. Nitrogen–helium gas mixtures were used to create low-disturbance jets having densities between 0.14 and 0.5 of the quiescent gas into which they were delivered. The experiments were designed to decouple the jet Reynolds number from the boundary layer thickness, and thereby examine whether the Reynolds number and momentum thickness were independent scaling parameters for global modes. This was accomplished by introducing extension tubes downstream of the nozzle contraction. It is believed that Re and D/θ_0 effects have not been found in earlier studies, since typical facilities follow similar trajectories in the (Re, D/θ_0)-plane.

The present results indicate that the onset of global instabilities in low-density jets with steep density gradients (top hat in the current study) is an independent function of S, D/θ_0 and Re. It was also apparent that the observation of global modes in low-density jets depends on the Re–D/θ_0 operating curve unique to the facility producing the jet, as does the maximum density ratio at which these modes might be observed. It was emphasized that to identify the critical density ratio, experiments will need to carefully traverse the (Re, D/θ_0)-plane. Finally, due to the globally unstable nature of this flow, the oscillating frequency was examined to determine whether a universal scaling could be identified, similar in principle to the classic St–Re curve observed in the Kármán vortex street. Using $S, D/\theta_0$ and Re as the governing independent parameters, the dimensionless frequency fD^2/ν was found to depend linearly on the group $Re(D/\theta_0)^{1/2}(1+S^{1/2})$. The significance of this scaling was conjectured to be due in part to the curvature of the jet column accompanying the strong self-excitation and closely connected to spatio-temporal stability theory. In particular, the satisfactory frequency scaling of the global mode frequency using the flow characteristics in the jet exit planes indicate that the frequency selection mechanism proposed by Pier & Huerre (2001) for wakes is appropriate to low-density jets having initially steep density profiles.

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